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UG – 094

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V Semester B.A./B.Sc. Examination, March/April 2021
(Semester Scheme) (CBCS) (F + R) (2016 – 17 and Onwards)
MATHEMATICS (Paper – VI)

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer* ***all*** *questions.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

   a) Write Euler's equation when the function f is independent of y.

   b) Show that the functional $\int_{x_1}^{x_2} (y^2 + x^2 y')\,dx$ assumes extreme values on the straight line y = x.

   c) Define Brachistochrone problem.

   d) Evaluate $\int_C xdy - ydx$, where C is a line $y = x^2$ from (0, 0) to (1, 1).

   e) Evaluate $\int_0^1 \int_0^2 (x + y)\,dy\,dx$.

   f) Evaluate $\int_0^1 \int_0^2 \int_1^2 xyz^2\,dx\,dy\,dz$.

   g) State Gauss divergence theorem.

   h) Evaluate by Stokes theorem $\oint_C yzdx + zxdy + xydz$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

   b) Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y\sin x)\,dx$ under the end conditions $y(0) = 0$; $y\left(\pi/2\right) = 0$.

OR

3. a) Solve the variational problem $\int_0^{\pi/2} (y^2 - y'^2)\,dx = 0$ with the conditions $y(1) = 0 = y(2)$.

   b) Define Geodesic. Prove that geodesic on a plane is a straight line.

P.T.O.

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Show that the extremal of the functional $\int_0^1 (y')^2 dx$ subject to the constraint $\int_0^1 y\, dx = 1$ and having $y(0) = 0$, $y(1) = 1$ is a parabolic arc.

OR

5. a) Find the extremal of the functional $\int_0^1 \left[(y')^2 + x^2\right] dx$ subject to the constraint $\int_0^1 y\, dx = 2$ and having end conditions $y(0) = 0$ and $y(1) = 1$.

b) Show that the extremal of the functional $\int_0^2 \sqrt{1+y'^2}\, dx$ subject to the constraint $\int_0^2 y\, dx = \pi/2$ and the end conditions $y(0) = 0$, $y(2) = 0$ is a circular arc.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Evaluate $\int_c (x+2y)dx + (4-2x)dy$ along the curve $C: \frac{x^2}{9} + \frac{y^2}{4} = 1$ in anticlockwise direction.

b) Evaluate $\int_c \left[(2x+y)dx + (3y+x)dy\right]$ along the line joining (0, 1) and (2, 5).

OR

7. a) Evaluate $\iint_R xy(x+y)\,dx\,dy$ over the region R bounded between the parabola $y = x^2$ and the line $y = x$.

b) Change the order of integration and evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2\, dy\, dx$.

8. a) Find the area of the circle $x^2 + y^2 = a^2$ using double integral.

b) Evaluate $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{a^2-x^2-y^2-z^2}}$.

OR

9. a) Evaluate $\iiint_R xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into cylindrical polar co-ordinates.

b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals.

PART – D

Answer **two full** questions : **(2×10=20)**

10. a) State and prove Green's theorem.

b) Using divergence theorem, evaluate $\iint_R \vec{F}.\hat{n}\,ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$.

OR

11. a) Verify Green's theorem in the plane $\oint_C (xy + y^2)dx + x^2\,dy$, where C is the closed curve bounded by $y = x$ and $y = x^2$.

b) Evaluate by Stokes theorem $\oint_C (\sin z\,dx - \cos x\,dy + \sin y\,dz)$, where C is the boundary of the rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.

12. a) By using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \le 4$ and $0 \le x \le 3$.

b) Evaluate $\iint \text{Curl}\,\vec{F}.\hat{n}\,ds$ by stokes theorem if $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z \le 2$.

OR

13. a) Verify Green's theorem for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by the line $x = 0$, $y = 0$, $x + y = 1$.

b) Verify the divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.
